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The first of these equations is the equation of the hodograph and the second is the vector equation of the trajectory.

The highest point reached by the projectile may be found by multiplying the first equation dotwise by $\boldsymbol{g}$ and setting $\boldsymbol{g}\cdot(d\boldsymbol{r}/dt) = 0$. Solving this equation for t we find $t = 1/k \log(1 - k\boldsymbol{v}_0\cdot\boldsymbol{g}^{-1})$, and this substituted in equations (1) and (2) gives the velocity and the position vector of the projectile at the highest point. Similarly, to find the time of flight and the range on the horizontal plane through the point of projection multiply the second equation by $\boldsymbol{g}\cdot$ and set $\boldsymbol{g}\cdot\boldsymbol{r} = 0$, solve for t, etc.

Since k is positive and t varies from zero to infinity e^{-kt} varies from 1 to 0 and the hodograph is seen to be the segment of a line which terminates at the tips of the vectors $\boldsymbol{v}_0$ and $\boldsymbol{g}/k$.

If the initial velocity is allowed to take all possible directions the hodographs will form a bundle of line segments extending from the tip of $\boldsymbol{g}/k$. If the initial speed is constant the segments will terminate in the surface of a sphere with center at the origin and radius v_0. If the angle of projection varies in a vertical plane and the initial speed is constant the segments will form a pencil one end of each segment resting on the tip of $\boldsymbol{g}/k$ (vertex of pencil) and the other on a circle of radius v_0. The vertex of the bundle or pencil corresponds to the limiting velocity which is the same for all initial velocities.

The cartesian equation of the trajectory is found from the vector equation in the usual way to be

$$y = \frac{g}{k^2}\log(1 - x/l) + \frac{mx}{l},$$

where

$$l = \frac{v_0 \cos\alpha}{k} \quad \text{and} \quad m = \frac{v_0 \sin\alpha}{k} + \frac{g}{k^2},$$

α being the angle of projection.

From this equation it is seen that the trajectory has an asymptote whose equation is $x = l$, which accords with the known fact that the trajectory of a body moving in any resisting medium has a vertical asymptote.

RECENT PUBLICATIONS.

REVIEWS.

CAJORI'S HISTORY OF MATHEMATICS.

A History of Mathematics. By FLORIAN CAJORI. New York, The Macmillan Company, 1919; pp. x + 514. Price $4.

It is twenty-five years since Professor Cajori, then already known for his contributions in the field of mathematical history, published the first edition of the work under review. During this period the book has been reprinted six times, and the author and publisher have now united in giving to students a second edition, to a considerable extent rewritten and in certain chapters materially enlarged.

The general plan of the first edition has been followed, the work being divided into three rather distinct parts, although not in any formal manner. The first part may be described as relating chiefly to the development of elementary mathematics and including the following main topics: the Babylonians, the Egyptians, the Greeks, the Romans, the Maya, the Chinese, the Japanese, the Hindus, the Arabs, and Europe during the Middle Ages. This part occupies 129 pages, or a little more than one fourth of the text. All things considered, this is a fair allotment of space.

The section relating to Europe during the sixteenth, seventeenth, and eighteenth centuries may be considered as the second part and may be described as concerned with the rise of higher mathematics. Naturally a large part of the work of these centuries was elementary, pertaining to the establishing of the present algebraic symbolism, to the discovery of the common theorems relating to equations, and to the better understanding of the nature of series; but the birth of higher mathematics, including modern higher geometry, the group theory, the general notion of functions, and the laying of scientific foundations, may also be said to have taken place in this period. To these centuries the author has assigned 148 pages, or somewhat more than 30 per cent of the text,—again a fair distribution of space.

The rest of the work is devoted to the period from 1800 to the present time, and includes 209 pages, or upwards of 43 per cent of the text. Roughly speaking, the space assigned to each of the three periods is in the ratio of 25 : 31 : 44, and for the purposes of students who will use this book the ratio is both significant and reasonable.

The first part of the work may be described as giving a racial view of the development of mathematics. This is natural and, for practical purposes, is necessary. Commerce had not yet come to the aid of scholars in the free transmission of thought, and racial characteristics predominated in science as they did in art, religion, and modes of government. In treating of Europe in the Middle Ages, however, Professor Cajori abandons the attempt to classify by races, tending apparently, and with some reason, to classify most of the scholars as churchmen and to consider Alcuin, Gerbert, Johannes Hispalensis, and Leonardo Fibonacci as belonging to the same branch of the human family tree.

In the second part of the work the tendency is to classify either by centuries or by periods which may be characterized by the influence of such men as Vieta, Descartes, Newton, and Euler. This is a natural and allowable treatment, although not the classification which some of those who give courses in the subject may prefer.

In the third part of the work the classification is by such subjects as synthetic geometry, analytic geometry, and algebra. Such a classification is, for the instructor in mathematics, the most helpful of all, and many readers will wish that Professor Cajori might have seen his way to extending it so as also to include the mathematical topics studied in the elementary school, the high school, and the junior college.

Taking the work as a whole, the classification of material is justifiable and the amount of information exceeds that to be found in any other general history of mathematics that has thus far appeared in English.

Even though one may read the book with a friendly eye it is probable that he will find himself giving expression to regret from time to time that Professor Cajori should have adopted certain forms of expression or have made certain definite assertions that are open to question. If a critic happens to have the audacity of youth, he will perhaps find himself giving a freedom to his pen that he will not approve in his later years. If he has himself yielded to the lure of authorship, he will remember Cowper's line,

> None but an author knows an author's cares,

and will seek to be more charitable. When we consider that the first volume of Cantor's monumental work appeared forty years ago, and that for the last twenty years there has been maintained in the third series of the *Bibliotheca Mathematica* a department devoted to a severe criticism of that great treatise, we realize how easy it is to find fault with any book. Indeed, it often seems as if it requires more genius on the part of a critic to refrain from petty faultfinding than it does to record a hundred slips of the memory, of the judgment, or of the pen.

The reviewer now proposes to mention certain typical excellencies of Professor Cajori's work, and then to mention certain types of defects, leaving the details of these defects for others to find, or for such helpful criticism as may be acceptable when another edition is under consideration.

Among the features which will render the work helpful to students of the history of mathematics there may first be mentioned the considerable bibliography given in the footnotes. Such assistance is valuable not merely in supporting some particular assertion of the author's but in showing to the student that there is an extensive literature awaiting him in case he cares to pursue the study further. It is not necessary that the books referred to should all be easily accessible, and in this case some of them are not; so long as they are worthy they act as a stimulus to further study, and on this account the list that has been given might profitably have been much longer.

Another feature that seems to this reviewer very fortunate is that of quoting from the original source in various cases in which the language is particularly significant. For this purpose Professor Cajori has assumed on the part of his readers some knowledge of French, Latin, and German, and it will not often be the case that students are so ignorant of at least two of these languages as not to profit by the quotations given. There is, however, a stronger reason for commending the plan, namely, that the student is thereby encouraged to feel that he is prepared to go to the original sources to secure first-hand information. This reviewer would have advised even more, much more, of this style of quotation, placing the material in footnotes so as not to disturb the reader who can not use it, and not hesitating to include easy Italian; but this is merely a matter of personal preference; the significant thing is that the step has been taken.

A third commendable feature is that of revealing the human side of those scholars whose names are mentioned, thus showing the history of mathematics as the history of men. Plutarch was none the less of a historian because he was the world's greatest biographer. Professor Cajori has given a great amount of information about people, and in the main this information has been well chosen.

When it comes to a consideration of types of those features which may be classed with "things one would rather have left unsaid," perhaps the first general criticism in the minds of most readers will relate to the style in which the book is written. "It is style alone," said the elder Disraeli, "by which posterity will judge of a great work, for an author can have nothing truly his own but his style." While this assertion is open to the criticism that applies to most epigrams, no one can read the treatise under review without sympathizing with Voltaire's advice, "In every author let us distinguish the man from his works." Professor Cajori does not talk as he too often writes. His habit of writing so many sentences in a kind of inverted order will probably annoy most of his readers. While often an aid to emphasis, or a desirable variant to one's usual style, or a means of connecting two sentences euphonically, this order is used so frequently by Professor Cajori as to detract from the clearness of the expression and to become an unfortunate mannerism. Thus we have within a few lines of each other, the sentences, "A profound scholar . . . and a mathematician . . . was Pierre de Fermat;" "A great contribution to geometry was his *De maximis et minimis;*" and "A contemporary mathematician . . . was Blaise Pascal." Sometimes this unfortunate habit even leads to temporary confusion, as in the sentence beginning, "About forty years after Archimedes flourished Apollonius . . .," in which the reader must go to the end of the sentence to find which name serves as the subject in the phrase quoted. Occasionally the skein becomes even more tangled, as in the sentence, "Greater taste than for geometry was shown by the Hindus for trigonometry"—a sentence which can easily be parsed, but which is nevertheless unfortunately expressed. Such constructions are an author's own property; he has a legal right to his own style, and a moral right as well. Nevertheless one cannot read the book without frequently being conscious of the remark of Disraeli's quoted above. It is a safe assertion that Professor Cajori himself would be greatly surprised to know the number of times this peculiarity of style asserts itself. Somehow, too, this reviewer does not connect the man who wrote this book with a sentence like the following; "We abstain from introducing additional Greek opinion regarding Egyptian mathematics, *or from indulging in wild conjectures.*" The sentence is perfectly grammatical, but it lacks the dignity which characterizes the author.

Somewhat related to the matter of style is the frequent use of a man's initials when there seems to be nothing gained. This is seen, for example, in the sentence (p. 222) relating to Jean Bernoulli: "Johann admired the merits of G. W. Leibniz and L. Euler, but was blind to those of I. Newton." If there were any other Leibniz, Euler, or Newton whom the reader would naturally have in mind, the use of the initials, while unusual, might be understood; but to see such initials with about half the names in the book is a matter of some annoyance. If the dates had been so frequently given with the names as to impress them on the mind, or to aid the reader to place certain men in proper chronological relation to others, there would have been a decided gain to counterbalance any annoyance arising from the appearance of the page; but one would as soon refer to W. Shakespeare and D. L. George as to I. Newton. One of the curiosities of this method of reference is seen in the case of P. Mersenne (p. 156) whose first name was Marin (p. 163), the P. evidently standing for Pater or Père.

It may be pardonable to mention also, in connection with the matter of style, the author's habit of referring to a writer whose biography has not already been given, or, indeed, to one who is quite unknown and who is never mentioned again. The student, who reads on page 32 what "H. G. Zeuthen finds," does not learn who H. G. Zeuthen is until he reads page 190; and when he reads (p. 44) the opinion of Venturi, he may search the book in vain to find who Venturi was or what his opinion may be worth. The same may be said of Thymaridas (pp. 59, 111), and a similar criticism may be made with respect to the Palatine Anthology, a work of which the student is quite certain to be ignorant. In this connection it may be suggested that it would have been better had "Thomas Finck, a native of Flensburg" (p. 151) been spoken of with reference to his nationality, since few readers will have the slightest idea in what country Flensburg is situated.

A second point of friendly criticism, for we should all be friendly in recognizing the amount of labor put upon this work, may properly relate to the statements of fact. Here the critic of the Eneström type will find more than this reviewer wishes were possible. A few of these statements will suffice to make clear the assertion that the book is not without its errors. That the Babylonian notation employed only two principles—"the additive and multiplicative" (p. 4) is itself contradicted on the same page in the statement that the subtractive principle was also employed. Indeed it should be said that the evidence of this fact is not confined by any means to the tablets found at Nippur. The frequent assertion that the Babylonians used sexagesimal fractions will be misunderstood; we have no evidence of such fractions before the Greeks introduced them into astronomy, even though the Babylonian use of sixty as a kind of radix in notation is well established, not merely by "two Babylonian tablets" but by many others. The fact that 30 sometimes meant $\frac{30}{60}$ does not illustrate the use of what the reader will understand by sexagesimal fractions, namely, degrees, minutes, seconds, thirds, and so on.

The date of Ahmes is, of course, still a matter of conjecture. When, however, it is given as "some time before 1700 B.C., "and when it next appears in the phrase "the incorrect formulæ of Ahmes of 3000 B.C.," and when it is next referred to as 3,000 years before Leonardo Fibonacci wrote his *Liber abaci* (1202 A.D.), it is safe to say that students will regret that certain assertions are made so positively.

The statement that "to Egypt Greece" is indebted for elementary geometry

(p. 15) is, of course, a matter of definition; but it will be misleading to the average reader until he has read five lines further on the page. The positive statement that Pythagoras sacrificed a hecatomb (p. 2) is not placed in doubt until the student has proceeded sixteen pages further in his reading. The statement that the method of exhaustion infers that the *circle* is exhausted (p. 23) is later corrected (p. 25) by the assertion that it is the "spaces between the polygons and circumferences" that are exhausted.

The statement that Plato did not acquire his taste for mathematics from Socrates (p. 25) is a rather strong one. In Dr. Jowett's delightful paraphrase of the *Theætetus* it is said that "he, Socrates, is a midwife, although this is a secret; he has inherited the art from his mother bold and bluff, and he ushers into light, not children, but the thoughts of men." Who knows how many thoughts of Plato were delivered by Socrates in this quasi-obstetric capacity?

That "Athens produced the greatest scientists and philosophers of antiquity" (p. 29) is a statement that will probably be criticized. As a matter of fact, it is always a cause for surprise that continental Greece produced so few scholars of this type in comparison with the islands of the Ægean Sea and with the Greek colonies.

Speaking of a space of 150 years, the statement that Sextus Julius Africanus was "an occupant of this long gap" sounds rather odd, although it is not probable that anyone will misinterpret it. That Pascal did not lead a "quiet and unaggressive life" (p. 163) will be an unpleasant surprise for most readers, and it would be interesting to know just what extended period of his life Professor Cajori had in mind in making the assertion.

Among the errors of fact there should be mentioned the subject of dates, already referred to in the case of Ahmes. It is not a matter of great importance to most readers whether Thales was born in 640 B.C. and died in 546 B.C., or whether the dates should be c. 640 and c. 542 respectively; and whether the eclipse which he prophesied took place in 585 B.C., or, as some writers assert, twenty-four years earlier. All that the general reader ordinarily cares for is an approximate date. Thus when the dates "580?–500? B.C." are given for Pythagoras, the purposes are sufficiently served. If, however, the dates for Pythagoras are queried, why should not the three dates for Thales also be given as doubtful? This is a work of reference and it is desirable that such statements of fact be accurate and that a doubt be expressed where one is generally supposed by good writers to exist. The same criticism applies in many cases throughout the book, including those relating to Anaxagoras, Archytas, Plato, Bede, Eratosthenes, Tabit ibn Korra, Hudde, and others.

As to the transliteration of Oriental names the author is no more at fault than most other historians. It is unfortunate that we have no board that can consider the whole matter with some semblance of authority and give us a standard system of diacritical marks and of the English equivalents of proper names as they appear in languages not using the Latin alphabet. If such forms as Ś, ṭ, ḥ, ī, and al- are to be used, as they are in this book, it is due to the reader to have a key to assist him in their pronunciation. This is especially the case where, as here, they are not used uniformly, as witness Madjritī (p. 104), Majrīti (p. 104), and Majriti (p. 109).

The final point which it seems worth while to mention is concerned with proofreading and printer's errors. It appears ungenerous to refer to such little matters as lack of uniformity, misspelled words, carelessness in the use of accents, and the like; and yet, like insect stings, in the aggregate they are, to say the least, causes for regret. For example, it is perfectly proper to use either "geometric" or "geometrical," but it is annoying to see no uniformity in the matter, even on a single page,—and similarly for "analytic" and "analytical." Is the *Elements* of Euclid to be considered as a single treatise hence taking a singular verb, or should we say "the *Elements* are . . ."? On page 30 the word is considered as plural and on page 32 as singular. Is Claudius Ptolemy (p. 46) to be preferred to Claudius Ptolemaeus (p. 45), or should we have merely Ptolemy (p. 47) or, as in other cases, C. Ptolemy?

Of printer's errors there is a much larger number than usual, but it is hardly the proper business of the reviewer to single them all out. They are simply annoyances, hardly attributable to the author. It is merely pressroom carelessness that leads to a form like $mc \gtrless nd$, with the equality sign omitted (p. 32), to the use of the angle symbol for inequality (p. 143), to spelling *nor* for *not* (p. 122), and to the use of Tivolis' for Tivoli's (p. 123), and Trigonometry for Trigonometrie (p. 137 n.). That the name of Apollonius is misspelled on page 36 is not the author's fault, primarily; it is the blunder of the proofreader. The spelling of paradoxies for paradoxes (twice on p. 23) is also an error of the press. It would be a pleasure to say the same for the name of Maupertuis, which is mispelled every time it appears. This is akin to placing Bagdad on the Euphrates (p. 99) instead of on the Tigris. The printer's attempts to make up the three peculiar Greek numerals (pp. 52, 53) are not altogether happy, but the error in his use of the zeta for 6 (p. 52) is quite unpardonable, not to speak of his use of the final sigma for the same purpose (p. 53). Śrīdhara is made to appear as S'rīdhara and Śiromani as S'iromani (p. 85), although a crude effort is later made to conform to the proper style by using the form S′rīdhara (p. 94).

The student will probably not be troubled by the uncertainty in the title of Hankel's *Geschichte* (pp. 10 n., 57 n.), but it would have been better had it been given correctly in each case.

The name Vlacq appears as Vlack in two cases, which is allowable except for uniformity; Āryabhata appears also (p. 85) as Āryabhata; "the Roman bishop Hippolytos" might perhaps better have the Roman termination to his name (p. 91); the degree sign is omitted in sin $(90 - \alpha)$ although given on the same page in sin $(90° - 2\gamma)$; the name Fine (Oronce) is correctly spelled on page 143 but appears incorrectly as Finé on page 116 (an error to which many of us must probably plead guilty); Coimbra appears as Coïmpre (p. 142 n.); Núñez or Nonius is called by the much less familiar name of Nunes (p. 142); Christopher Clavius appears as Christophorus Clavius (p. 144), a form which is rather unusual in English works, and also in the curious guise of C. S. Clavio (p. 184.) Mention should also be made of the genitive forms of certain proper names. Shall we expect Wallis' or Wallis's, where the name is not a plural form? There is authoritative sanction for either, but there is no sanction for using the two forms indiscriminately as is the case with various names throughout this work.

With respect to the index there should be mentioned the fact that it is unusually complete and that it has one decided improvement over most efforts of this kind. This improvement consists in giving the most important reference first, and in the use of subtitles under the heading entries, each of which is a great convenience. There are various omissions, such as Coss, Harun al-Rashid, Fine (O., but see Orontius), and Salviati, and there are a few misspelled names, like Durege for Durège, but on the whole there is no reasonable cause for complaint.

In closing this review it should be said once more that the purpose of the writer has not been to find fault with respect to small details. He has endeavored to call attention to the distinctively good features of the work and to state only the leading features that are likely to be subjects of adverse criticism. If the proper limits of a review of this kind had permitted, he would have been glad to discuss such questions as the Roman use of 120 as a diameter, the rise of the sexagesimal fraction, and the causes of the prominence of mathematics in various epochs and in various countries, and to commend specifically certain other features of value. He wishes, however, to add the statement that the book shows a wide reading of secondary material and a considerable study of original sources. It will, of course, have a place in every mathematical library of any importance and will prove a helpful work of reference, particularly with respect to the mathematics of the last two centuries.

DAVID EUGENE SMITH.

Report of the President of the Carnegie Institution of Washington for the year ending October 31, 1919. Washington, D. C., 1919. 4to. 37 pp.

For the mathematician perhaps the most interesting items in this report by President R. S. WOODWARD, a charter member of the Association, are the paragraph concerning the Hooker telescope on Mount Wilson (pages 16–17) which "under repeated tests has proved efficient quite beyond the conservative theoretical predictions of attainable capacities," and the following paragraph (pages 18–19):

> "The most elementary, the most essential, and hence the most widely used, if not esteemed, of the sciences is arithmetic. It is a fundamental requisite, in fact, of all exact knowledge. Ability to add, subtract, multiply, and divide affords probably the simplest test of capacity for correct thinking. Conversely, inability or indisposition to make use of these simple operations affords one of the surest tests of mental deficiency, as witnessed, for example, by numerous correspondents who are unable to or who refuse to apply these operations to the finances of the Institution. But the familiar science of arithmetic lies at the foundation also of a much larger and a far more complex structure called the theory of numbers. This theory has been cultivated by many of the most acute thinkers of ancient and modern times. It has more points of contact with quantitative knowledge in general than any other theory except the theory of the differential and